Demand analysis and elasticity: classification of goods and consumer behavior

Key concepts

- **Demand functions:** Represent the relationship between the quantity demanded of a good and variables such as its own price, the price of related goods, and consumer income
- **Partial derivatives:** Allow us to analyze the marginal effect of a change in one variable (price or income) on the quantity demanded, while holding the others constant
- **Elasticity:** Measures the sensitivity of quantity demanded to percentage changes in price or income, helping to classify goods according to their behavior

Classification of goods

1. According to own price

- **Typical goods:** When the price of the good increases, the quantity demanded decreases. Most goods behave this way:

$$\frac{\partial Q}{\partial P} < 0$$

- **Giffen goods:** In exceptional situations, an increase in the good's price leads to an increase in its demand, due to combined substitution and income effects. These should not be confused with Veblen goods, which are luxury goods demanded for their status-signaling characteristics:

$$\frac{\partial Q}{\partial P} > 0$$

Intuition: Under normal conditions, the inverse relationship (higher price, lower demand) is expected; Giffen goods represent rare cases where, due to budget constraints, a price increase leads to greater consumption of the good.

2. According to the price of another good

- **Substitute goods:** If an increase in the price of one good leads to an increase in the demand for another, the goods are considered substitutes. This occurs when the consumer switches to the relatively cheaper good. For example, rival brands like Coca-Cola and Pepsi:

$$\frac{\partial Q_1}{\partial P_2} > 0$$

- **Complementary goods:** If an increase in the price of one good reduces the demand for the other, they are classified as complements, since they are consumed together. For example, coffee and sugar:

$$\frac{\partial Q_1}{\partial P_2} < 0$$

Intuition: The existence of substitutes allows the consumer to switch products when the price of one increases, while complementary goods are jointly consumed to satisfy a need.

3. According to income

- **Normal goods:** Demand increases as consumer income rises. Most goods tend to be normal:

$$\frac{\partial Q}{\partial I} > 0$$

- **Inferior goods:** Demand decreases as income rises, since consumers opt for higher-quality goods. For example, second-tier brands:

$$\frac{\partial Q}{\partial I} < 0$$

Intuition: Classification by income reflects the consumer's ability to adjust their consumption habits based on purchasing power.

Demand elasticity

Elasticity quantifies how sensitive the quantity demanded is to changes in prices or income.

Price and income elasticity

$$\text{Price elasticity} = \frac{\Delta Q/Q}{\Delta P/P} = \frac{P}{Q}\frac{\partial Q}{\partial P}$$

- **Interpretation (in absolute terms):**
  - If
  $$\left|\frac{P}{Q}\frac{\partial Q}{\partial P}\right| > 1$$
  demand is **elastic** (sensitive to price changes)
  - If
  $$\left|\frac{P}{Q}\frac{\partial Q}{\partial P}\right| < 1$$
  demand is **inelastic** (not very sensitive)
  - If
  $$\left|\frac{P}{Q}\frac{\partial Q}{\partial P}\right| = 1$$
  elasticity is **unitary** (demand changes in the same proportion as price)
  - If
  $$\frac{\partial Q}{\partial P} = 0 \quad (\text{which implies } \left|\frac{P}{Q}\frac{\partial Q}{\partial P}\right| = 0)$$
  demand is **perfectly inelastic** (quantity demanded remains constant regardless of price changes)

- Although the concept is presented as **price elasticity**, this idea extends to **income elasticity** and any other function that admits partial derivatives. For example, income elasticity is defined as:

$$\text{Income elasticity} = \frac{\Delta Q/Q}{\Delta I/I} = \frac{I}{Q}\frac{\partial Q}{\partial I}$$

  where I represents income

Luxury goods vs. essential goods

The classification of goods as luxury or essential is based on **income elasticity**, which measures the sensitivity of quantity demanded to changes in the consumer's income. It is important to note that this classification applies only to **normal goods**.

Essential goods (basic needs)

For essential goods, the income elasticity falls within the range:

$$0 < E_I < 1$$

where

$$E_I = \frac{I}{Q}\frac{\partial Q}{\partial I}$$

This means that a percentage increase in income leads to a smaller percentage increase in quantity demanded. These goods satisfy basic needs, and their consumption tends to stabilize even as income rises.

Luxury goods

In contrast, luxury goods are characterized by:

$$E_I > 1$$

This implies that a percentage increase in income generates a larger percentage increase in demand. These goods are purchased in greater quantities when consumers have more resources, reflecting more discretionary spending behavior.

Examples:

- **Essential goods:** Basic foods (e.g., bread and rice), generic medications, housing
- **Luxury goods:** Designer clothing, high-end cars, fine jewelry

Example

Suppose the demand function for a good is given by:

$$Q(P, I) = 200 + 0.3I - 4P$$

where P is the price of the good and I is the consumer's income.

Partial derivatives:

$$\frac{\partial Q}{\partial P} = -4, \quad \frac{\partial Q}{\partial I} = 0.3$$

The negative sign in $\frac{\partial Q}{\partial P}$ indicates that as price increases, quantity demanded decreases (typical good). The positive sign in $\frac{\partial Q}{\partial I}$ shows that as income increases, demand grows—characterizing the good as normal.

Evaluation at a specific point:

Let $P = 15$ and $I = 500$. Then:

$$Q(15, 500) = 200 + 0.3 \times 500 - 4 \times 15 = 200 + 150 - 60 = 290$$

Price elasticity calculation:
Price elasticity is defined as:

$$E_P = \frac{P}{Q}\frac{\partial Q}{\partial P}$$

Substituting the values:

$$E_P = \frac{15}{290} \times (-4) \approx -0.207$$

In absolute value, $|E_P| \approx 0.207$, which indicates that demand is **inelastic** (not very sensitive to price changes).

Income elasticity calculation:

$$E_I = \frac{I}{Q}\frac{\partial Q}{\partial I}$$

Substituting the values:

$$E_I = \frac{500}{290} \times 0.3 \approx 0.517$$

This means that a 1% increase in income leads to approximately a 0.517% increase in quantity demanded—the demand is also inelastic with respect to income.

Interpretation regarding luxury vs. essential goods:
Since $0 < E_I < 1$, the good is classified as a **essential good** (or basic need), as the demand response to changes in income is relatively low.